Acoustic Fourier Transform

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The fast Fourier transform was utilized to examine the behavior of mathematical, generated, and natural sound waves in frequency space. Using a mathematical sine wave, it was determined the linewidth of the fundamental frequency decreases as the number of cycles and time range of the original wave increases. Analysis of a sine and square wave revealed that harmonics above the fundamental frequency change the sound quality. Two tuning forks of the same note played together produced a beat frequency of 0.681 ± 0.002 Hz. The timbre of a whistle as well as the vowel sounds "O" and "E" were compared. The harmonics of a guitar and piano were analyzed and revealed that the piano had 9 harmonics of higher relative amplitude while the guitar had 4 harmonics with only one of high relative amplitude. The harmonic frequencies were determined to change the timbre and sound quality of a note.

I. INTRODUCTION

The purpose of this experiment was to practice and utilize the fast Fourier transform (FFT) to analyze mathematical waves and to explore the properties of harmonics and fundamental frequencies in natural sound waves. A sound wave is produced by creating pulses of high and low density air particles using high and low pressure. When measuring a sound wave, most instruments record the intensity of the wave over time. However, it can be valuable to analyze the intensity of the wave with respect to its frequency. To transform a wave from time space to frequency space, a Fourier transform can be utilized. The integral form of the Fourier transform is given by Equation 1:

$$B(f) = \int_{-\infty}^{\infty} A(t)e^{-i2\pi f t}dt \qquad \text{Eq. (1)}$$

Where *B(f)* is the transformation of function *A(t)* in frequency space, *A(t)* is a function in time space, *i* is the imaginary number, *f* is the frequency, and *t* is time. However, most software programs utilize a modified version of the Fourier transform called the fast Fourier transform. If the number of points is equal to 2^n, such as 2048, the FFT can be calculated and scaled using Equation 2:

$$\delta f = \frac{1}{\Delta t} \qquad \text{Eq. (2)}$$

Where δf is the change in frequency between points and Δt is the total range of the time axis. The produced FFT graph is mirrored along the x-axis. When the FFT of a wave is computed, a peak intensity will be shown at the fundamental frequency of the wave. Some waves have harmonics, which will appear as smaller-amplitude peaks in the FFT graph.

II. APPARATUS

The apparatus consisted of the following:

- Radioshack microphone 3303043
- Preamplifier, Rainbow Labs AA-1

- Tektronix TBS 1052B Digital Oscilloscope
- Altec Lansing Speaker
- Cenco tuning forks, A and C
- Sargent Welsh Tuning forks, A and C
- Instek Function Generator GFG-82198
- Guitar
- Electric piano
- Microphone
- FFT Software (Excel, MATLAB)

III. PROCEDURES AND RESULTS

A. Performing Mathematical FFT

In this section, the FFT would be applied to a perfect sine wave to better understand the properties of the transform. The sine wave studied in this section is summarized by Equation 3:

$$y(t) = \sin(2\pi * 100t) \qquad \text{Eq. (3)}$$

According to this structure of sine wave, the fundamental frequency is expected to be around 100 Hz. To begin, the sine wave was plotted from 0 to 0.1 seconds using 2048 points in MATLAB. The FFT with a Hamming filter was applied to the graph and scaled according to Equation 2. The sine wave and its transform are displayed in Figure 1. The FFT revealed a fundamental frequency at 100 Hz with an amplitude of 102.5 (units). The full width at half maximum was measured to be 10 Hz. The full range of frequency on the FFT graph was 1024 Hz.

Next, the same sine wave was plotted over a longer period of time, 0 to 1.0 seconds. The FFT was computed as before and the frequency axis adjusted to focus upon the fundamental frequency. As before, the fundamental frequency was observed at 100 Hz and there were no harmonics. However, the amplitude of the fundamental frequency was 1020 (units). The full width at half maximum was measured to be 1 Hz. The full range of frequencies on the FFT graph was once again 1024 Hz.

The two graphs were compared. For the shorter sine wave from 0 to 0.1 seconds, 10 cycles of the waveform were recorded. For the longer wave from

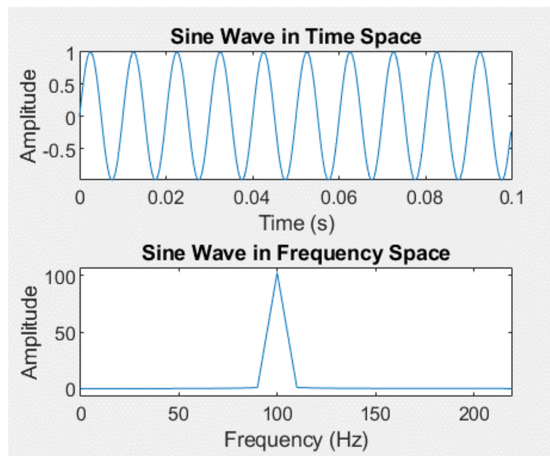

Fig. 1. Sine Wave in time space and frequency space, with frequency axis adjusted to focus on the fundamental frequency peak.

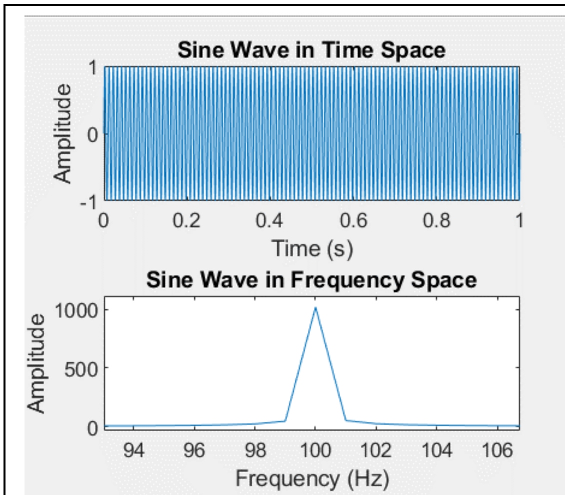

Fig. 2. Sine wave in time space and frequency space, with frequency axis adjusted to focus on the fundamental frequency peak, for longer time range.

0 to 1.0 seconds, 100 cycles were recorded. The FFTs of the two waveforms are graphed together in Figure 3. This experiment demonstrated that as the number of cycles increases, the range of frequency remains constant. However, the amplitude of the frequency peaks increases as cycles increase and with the length of time recorded. The linewidth, conversely, decreases as the time interval increases. From these relationships, it can be concluded that high precision FFT requires a high number of collected cycles, and large frequency range FFT requires a longer time interval.

B. Analyzing Sine and Square Waves with Function Generator

The next section of the experiment analyzed the FFT of a sine and square wave generated by a function generator. To begin, the function generator was connected to the Channel 1 of the oscilloscope and both devices were turned on. The function generator was set to produce a 1 kHz sine wave. Trigger and horizontal scale was adjusted on the oscilloscope to ensure that the wave was not being under-sampled and a consistent, constant waveform was visible. Data from this waveform was captured and saved for analysis. The number of points was truncated to 2048 to allow for the software to compute the FFT. The data was imported to MATLAB and the FFT computed. The resulting wave is shown in Figure 4.

The peak frequency of the sine wave was 977 Hz, with an amplitude of 638.7 V/s and a linewidth of 244.1 Hz. There were no peaks beside the fundamental frequency of the wave.

Next, a 1kHz square wave was generated and recorded as before. The resulting wave and its transform are shown by Figure 5.

The peak frequency of the square wave was 977 Hz with an amplitude of 849.9 V/s. The linewidth of the fundamental frequency was 244.1 Hz. Unlike the sine wave, multiple harmonic frequencies were recorded by the FFT. The harmonic peaks occurred around 3 kHz, 5 kHz, 7 kHz, and so on with decreasing amplitude. These small harmonic peaks continue at these increments along the entire frequency range.

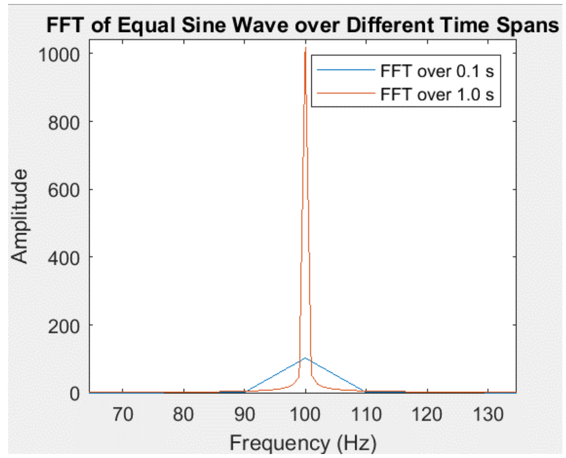

Fig. 3. FFT of the same sine wave over different time spans, where blue corresponds to 0.1 s and orange corresponds to 1.0 s.

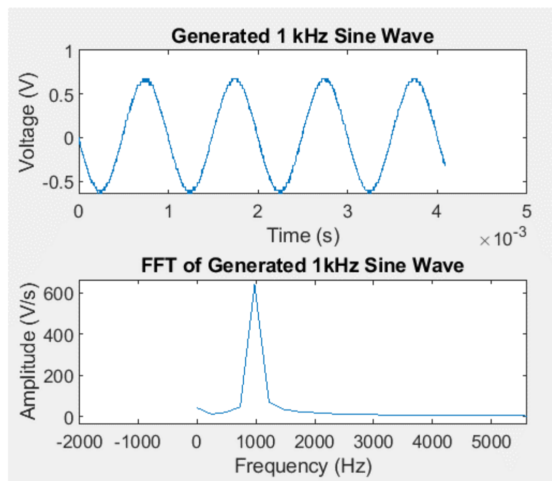

Fig. 4. Function generator sine wave in time space and frequency space.

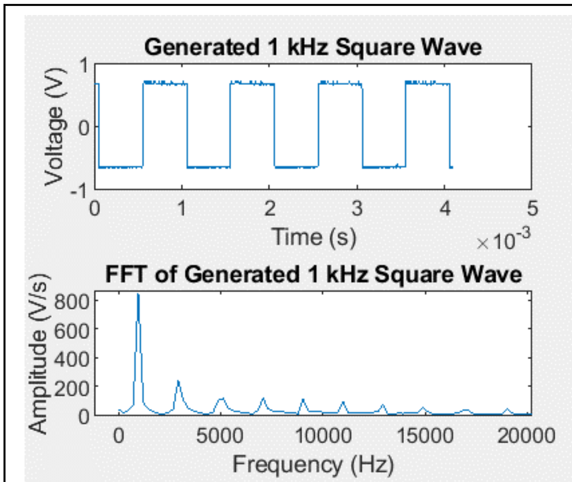

Fig. 5. Function generator square wave in time space and frequency space.

The function generator and oscilloscope were connected to a speaker. Using the same settings as before, a 1 kHz sine wave was played. It had steady, single-note sound. Next, a 1 kHz square wave was played. It had a fuller sound with harmonics creating a richer timbre. This aligns with the behavior for the sine and square wave recorded by the FFT.

C. Tuning Forks, Whistle, and Human Voice

The next section of the experiment was dedicated to recording and analyzing real-world sounds. To begin, two tuning forks of the same pitch were selected. For this experiment, the note C was selected. The oscilloscope time scale was set to a time base around 0.5 seconds per square. A microphone was connected to Channel 1 of the oscilloscope and held next to the tuning forks as they were both played simultaneously. The recorded waveform was saved to Excel and is showed by Figure 6.

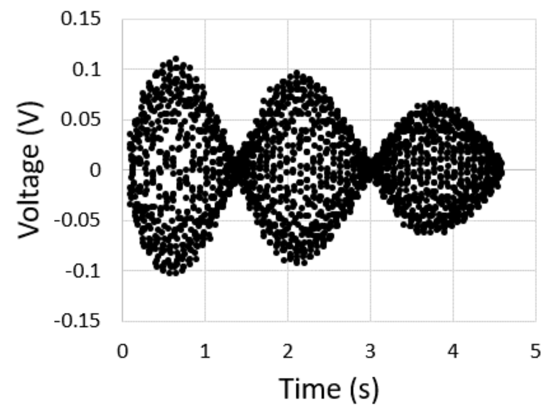

Fig. 6. Beats observed when two tuning forks of same pitch were played simultaneously.

The phenomenon recorded through this process is beating. When two notes of very close frequency are played together, their sound waves interfere. Since they have very similar frequencies and periods, they oscillate between constructive interference and destructive interference. The peaks of the beats occur when the two waves constructively interfere, which is to say that they combine to form a peak with a larger amplitude than either of the individual waves. However, the waves also destructively interfere when they are out of phase, resulting in their amplitudes cancelling out to produce a near-zero amplitude in the combined waveform [1]. From this phenomenon, beats are observed as shown in Figure 6.

The frequency of the combined waveform shown in Figure 6 was measured to be 0.681 ± 0.002 Hz. This frequency describes the difference in frequencies between the two waveforms [1]. In other words, the two tuning forks used in this experiment were 0.681 ± 0.002 Hz apart. This discrepancy could be due to the manufacturing standards of the companies that developed the tuning forks.

Next, the waveform of a whistle would be analyzed. The oscilloscope scale was set to a time base of 0.025 seconds per square. Using the microphone connected to Channel 1 of the scope, a lab member whistled into the microphone and the waveform was saved for analysis. The waveform and its FFT are shown by Figure 7.

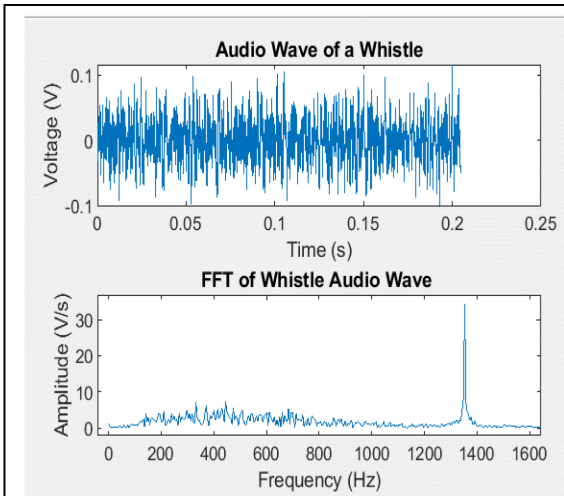

Fig. 7. Recorded audio wave and FFT of a human whistle.

The fundamental frequency of the whistle was 1353 Hz with an amplitude of 34.14 V/s. There are irregular harmonics in the lower frequency range.

If two people's whistles were compared, there would be differences in the timbre that would translate to a different arrangement of harmonics in frequency space. Instead of lower harmonics as recorded in this lab, someone may be able to produce whistles with harmonics at a frequency higher than the fundamental frequency. As everyone's anatomy varies slightly, the whistle each individual can produce is unique in timbre and therefore would have a different audio signature in frequency space.

Whistling works by using the shape of the mouth to produce pulses of high and low pressure. Air is pushed towards the lip, causing a buildup of air molecules that create a high-pressure region. When the air is allowed to release, the molecules have a higher velocity than the stagnant air around it, causing a ripple of energy that travels as a sound wave [2].

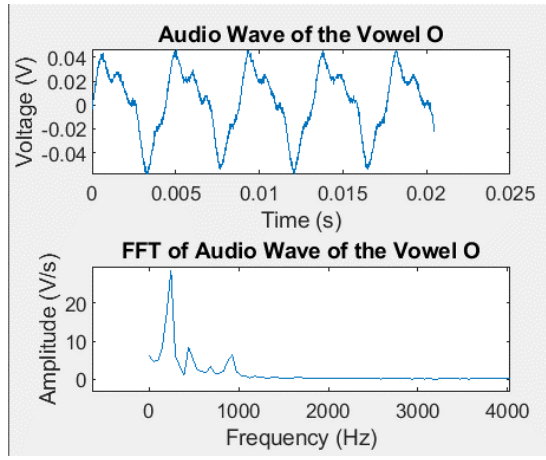

Fig. 8. Recorded audio of a human speaking the vowel sound "O" and its respective FFT.

Finally, a human voice speaking a vowel sound was recorded using the microphone connected to Channel 1 of the oscilloscope. The oscilloscope time base was kept at 0.025 seconds per square. The vowels "O" and "E" was recorded and analyzed in MATLAB. Figure 8 shows the audio wave analysis for the vowel "O" while Figure 9 shows the audio wave for the vowel "E".

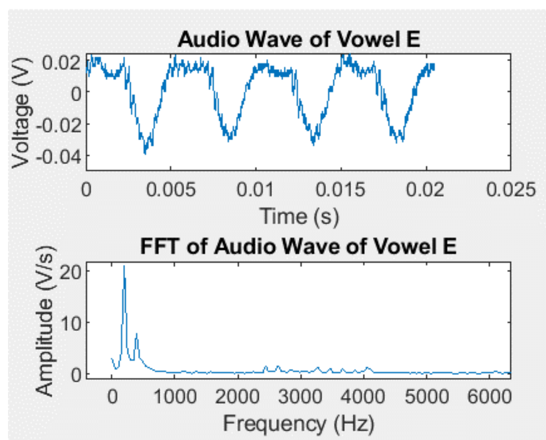

Fig. 9. Recorded audio of a human speaking the vowel sound "E" and its respective FFT.

For the vowel "O", the fundamental frequency was recorded to be 244.3 Hz with an amplitude of 28.5 V/s. There were several small harmonics, measured at 439.7 Hz, 683.9 Hz, and 928.2 Hz.

For the vowel "E", the fundamental frequency was 195.4 Hz with an amplitude of 21 V/s. There were several more harmonics measured at 390.8 Hz, 2443 Hz, 2638 Hz, 3273 Hz, 3468 Hz, 3664 Hz, 3859 Hz, and finally 4055 Hz.

Compared to the FFT for the vowel O, the FFT for the vowel E had many more harmonics. Although this could not be quantitatively measured, the sounds with more harmonics were consistently noted to have a richer timbre than those that were recorded with fewer harmonics. A qualitative correlation was suggested between the quality of sound and the number of harmonics of the audio wave in frequency space.

D. Harmonics with Guitar and Electric Keyboard

The last section of this experiment studied the harmonics produced by instruments, in this case a guitar and an electric keyboard. Similar to the previous setup, the microphone was connected to Channel 1 of the oscilloscope. A tuned note, in the case a D flat, was played with the guitar into the microphone and saved from the oscilloscope. The resulting audio wave is shown in Figure 10.

The waveform from the guitar had a fundamental frequency of 139.3 Hz. Harmonics were measured at 278.456 Hz, 417.684 Hz, 556.913 Hz, and 837.811 Hz. The harmonic number, which describes the multiple of the fundamental frequency that describes the harmonic, as well as their amplitudes are summarized in Table I. The fundamental frequency is bolded for clarification.

Table I: Harmonic Analysis for Guitar		
Frequency (Hz)	Amplitude (V/s)	N
139.228	**35.2082**	1
278.456	3.44235	2
417.684	16.1106	3
556.913	3.11617	4
837.811	1.21183	6

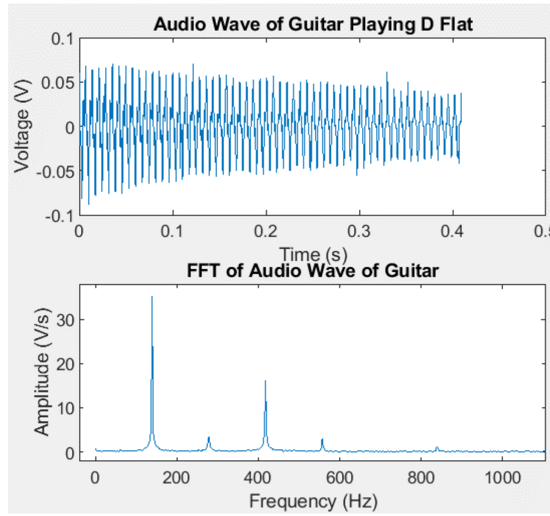

Fig. 10. Recorded audio wave of a guitar playing a D flat.

The third harmonic had the largest amplitude compared to the others.

Next, an electric piano played the same note, D flat, into the microphone. The audio wave was saved from the oscilloscope and the resulting waveform and its transform are shown in Figure 11.

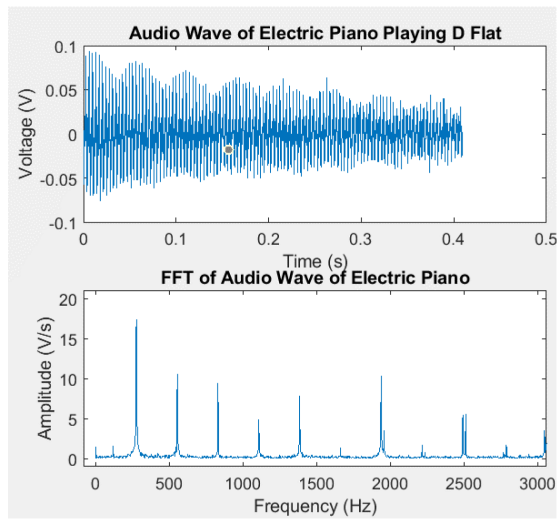

Fig. 11. Recorded audio wave of an electric piano playing a D flat.

The electric piano had a fundamental frequency of 276.014 Hz and almost ten harmonics. The harmonics and their respective amplitudes and number are recorded in Table II.

Table II: Harmonic Analysis for Piano		
Frequency (Hz)	Amplitude (V/s)	N
276.014	**17.3536**	1
554.47	10.5866	2
830.484	9.431	3
1106.5	4.9043	4
1384.95	7.8707	5
1660.97	1.34016	6
1936.98	10.2996	7
2215.44	1.71941	8
2491.45	5.5137	9

The piano sound had 9 audible harmonics in total, producing a very clear, high-quality sound.

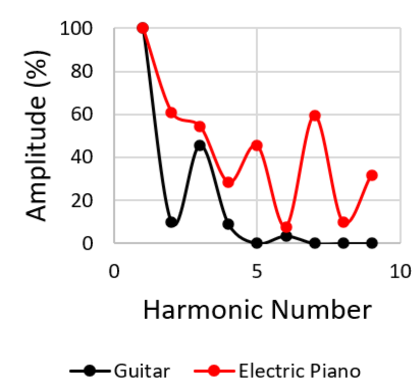

Fig. 12. Recorded audio wave of an electric piano playing a D flat.

Next, a plot describing the relative amplitudes of the harmonics for the two instruments was generated, as shown in Figure 12.

The most intense peak of each instrument was scaled to 100 to ease comparison, such that the amplitudes were presented as a percentage of the maximum amplitude recorded for each instrument. From this plot, it is clear that the piano had more harmonics with a higher relative amplitude, whereas the guitar only had one harmonic with a relative amplitude larger than 50%. These differences in the harmonic behavior of their waveforms helps to characterize the differences in timbre and sound observed between different musical instruments.

IV. SUMMARY

This experiment yielded valuable insights into the nature of the fast Fourier transform and how sound quality relates to harmonics. The first section of this experiment analyzed a mathematical sine wave to study the behavior of the fast Fourier transform. It was determined that when a longer time frame of a wave and therefore more cycles of the wave are recorded, the linewidth of the fundamental frequency peaks decreases. However, the amplitude of the frequency peaks will increase with the length of time and number of cycles recorded in time space. From this data, it was determined that the precision of an FFT can increase if data is recorded with more cycles or over a longer period of time.

The next section compared the behavior of a sine wave and a square wave both set at the same frequency 1 kHz. From the FFT analysis, it was determined that the square wave had a set of harmonics that built upon the fundamental frequency whereas the sine wave did not. The audio of the square wave indicated that the presence of harmonics can alter the timbre of sound and therefore improve the richness and sound quality of the note.

Next, real-world sounds were recorded. The beat phenomenon was successfully demonstrated with two tuning forks. The beat frequency was 0.681 ± 0.002 Hz, indicating that the two tuning forks differed in played frequency by 0.681 ± 0.002 Hz. The audio wave of the whistle had uneven, irregular harmonics below the fundamental frequency. The audio of two vowel sounds, “E” and “O” were compared. It was determined that the “E” sound had more harmonics as well as a steadier sound. These results again indicated that the presence of harmonics affects the timbre and sound quality of a note.

Finally, the harmonic behavior of a guitar and electric piano playing D flat were compared. The piano had 9 harmonics with higher relative amplitudes, whereas the guitar had 4 measured harmonics and only the third harmonic had a relative amplitude more than 50% of the amplitude of the fundamental frequency. Overall, this experiment demonstrated the importance harmonics in the timbre and sound quality of a played note, which helps to differentiate the sound of different instruments.

There were several sources of error in this experiment. The most prevalent was the background noise of the lab space where this experiment was conducted. Other lab groups were working in the same space with no sound insulation, and so background noise was likely picked up by the microphone and would have affected the baseline for results. Attempts were made when recording data to only record when the room was relatively quiet, but this does not eliminate the effect of background noise entirely. A source of error for the comparison of the two instruments was the tuning. The guitar was tuned to a fingered D flat note using an app. However, it should be noted that this tuning was relative to A 440 and the app did not provide a precise margin of error. It was possible that the D flat played by the piano and the guitar were not equal in frequency. Overall, the results could be improved by adding sound proofing to the space and using accurately tuned instruments.

V. REFERENCES

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[2] American Whistle Co. (n.d.). *How a Whistle Works*. American Whistle Co. Retrieved June 26, 2022, from https://www.americanwhistle.com/how#:~:text=Air%20enters%20the%20whistle%20at,the%20length%20of%20the%20whistle